J Vis (2015) 18:409–412
DOI 10.1007/s12650-014-0249-7

SHORT PAPER

Nao Ninomiya · Yamato Kubo · Daisuke Barada · Tomohiro Kiire

Direct measurement of longitudinal velocity by Doppler phase-shifting holography

Received: 14 October 2014 / Revised: 5 November 2014 / Accepted: 6 November 2014 / Published online: 6 December 2014

1 Introduction

According to the development of computer and imaging devices, the new measuring techniques based on flow visualization have been developed and utilized in many kinds of field. The typical examples are so-called PIV, i.e., particle image velocimetry, and PTV, i.e., particle tracking velocimetry. The principal setup of PIV utilizes one high-resolution camera and a laser light sheet, which illuminates the tracer particles suspended in the fluid flow. By calculating the displacement of particles, the two-dimensional distribution of the two-dimensional velocities can be obtained. Recently, by implementing several cameras and the careful calibration of their three-dimensional orientations, three-dimensional measurement of the three-dimensional velocities can also be achieved by so-called Tomographic PIV.

In order to understand the details of the flow field in the micro- and nano-fluidic devices, it is necessary to measure the three-dimensional velocity under a microscopy. Thus, the development of a new measuring technique for the three-dimensional velocity by a single camera observation is strongly needed. One solution is the use of holography (Schnars and Jueptner 2005), but it has been known that the resolution in the depth direction is very poor for the commonly used in-line holography. Presently, the Doppler phase-shifting holography (Kikuchi et al. 2010) has been used for the three-dimensional measurement of an object. This method extracts the signal of a fixed frequency caused by the Doppler beat between the object light and the reference light. The frequency of the beat is determined by the velocity difference between the object light and the reference light. This implies that the velocity of an object can be measured by the beat frequency. This implies that the longitudinal velocity, which cannot be obtained by the conventional measurement, can be measured together with the two-dimensional in-plane velocities that can be easily measured by PIV or PTV. If this method is applied to the particles motions, three-dimensional velocities can be measured by a single camera.

2 Doppler phase-shifting holography

Digital holography observes the interference of the observed light scattered at the surface of an object and the reference light. Then, the three-dimensional shape of an object can be reproduced by calculating the diffraction by the computer. But, in the commonly used in-line holography, the diffraction is contaminated

---

N. Ninomiya (✉) · Y. Kubo · D. Barada · T. Kiire
Graduate School of Advanced Interdisciplinary Science, Utsunomiya University, 7-1-2 Yoto,
Utsunomiya 321-8585, Japan
E-mail: nino@utmu.jp
Tel.: +81-28-689-6030

by the 0th order object light and the accuracy in the depth direction is deteriorated. In order to improve this drawback, several phase-shifting methods have been proposed, which can calculate only the 1st order diffraction and thus the longitudinal accuracy is guaranteed. Presently, Doppler phase-shifting holography is chosen because it is capable of unsteady measurement and does not need multiple cameras. Figure 1 shows the experimental setup of a Doppler phase-shifting holography using Michelson Interferometry.

The complex intensities of the object light and of the reference light are expressed as follows:

$$E_{\mathrm{O}}(x,y,t) = a_{\mathrm{O}}(x,y)\exp\{\mathrm{i}[\phi_{\mathrm{O}}(x,y) - \omega_{\mathrm{O}}(t)t]\} \tag{1}$$

$$E_{\mathrm{R}}(x,y,t) = a_{\mathrm{R}}(x,y)\exp\{\mathrm{i}[\phi_{\mathrm{R}}(x,y) - \omega_{\mathrm{R}}(t)t]\} \tag{2}$$

where a_{O} and a_{R} represent the amplitudes, ϕ_{O} and ϕ_{R} are the phase angles and ω_{O} and ω_{R} are the angular velocities of the object light and the reference light. If the object travels at velocity υ_{O} and the reference mirror at υ_{R}, the angular velocities are shifted by the Doppler effect of light as:

$$\omega_{\mathrm{O}}(t) = \omega_0\left(\sqrt{\frac{1+\upsilon_{\mathrm{O}}(t)/c}{1-\upsilon_{\mathrm{O}}(t)/c}}\right)^2 \cong \omega_0\left(1+\frac{2\upsilon_{\mathrm{O}}(t)}{c}\right) \tag{3}$$

$$\omega_{\mathrm{R}}(t) = \omega_0\left(\sqrt{\frac{1+\upsilon_{\mathrm{R}}(t)/c}{1-\upsilon_{\mathrm{R}}(t)/c}}\right)^2 \cong \omega_0\left(1+\frac{2\upsilon_{\mathrm{R}}(t)}{c}\right) \tag{4}$$

where ω_0 is the angular frequency of the light source and c is the speed of light. The Doppler effects are doubled by the mirror and the object when they receive the light and when they reflect the light. Consequently, the superposition intensity of the holograms detected by the image sensor is expressed as follows:

$$\begin{aligned} I(x,y,t) &= |E_{\mathrm{O}}(x,y,t) + E_{\mathrm{R}}(x,y,t)|^2 \\ &= a_{\mathrm{O}}^2(x,y) + a_{\mathrm{R}}^2 + 2a_{\mathrm{O}}(x,y)a_{\mathrm{R}}\cos[\Delta\phi(x,y) - \Delta\omega(t)t] \end{aligned} \tag{5}$$

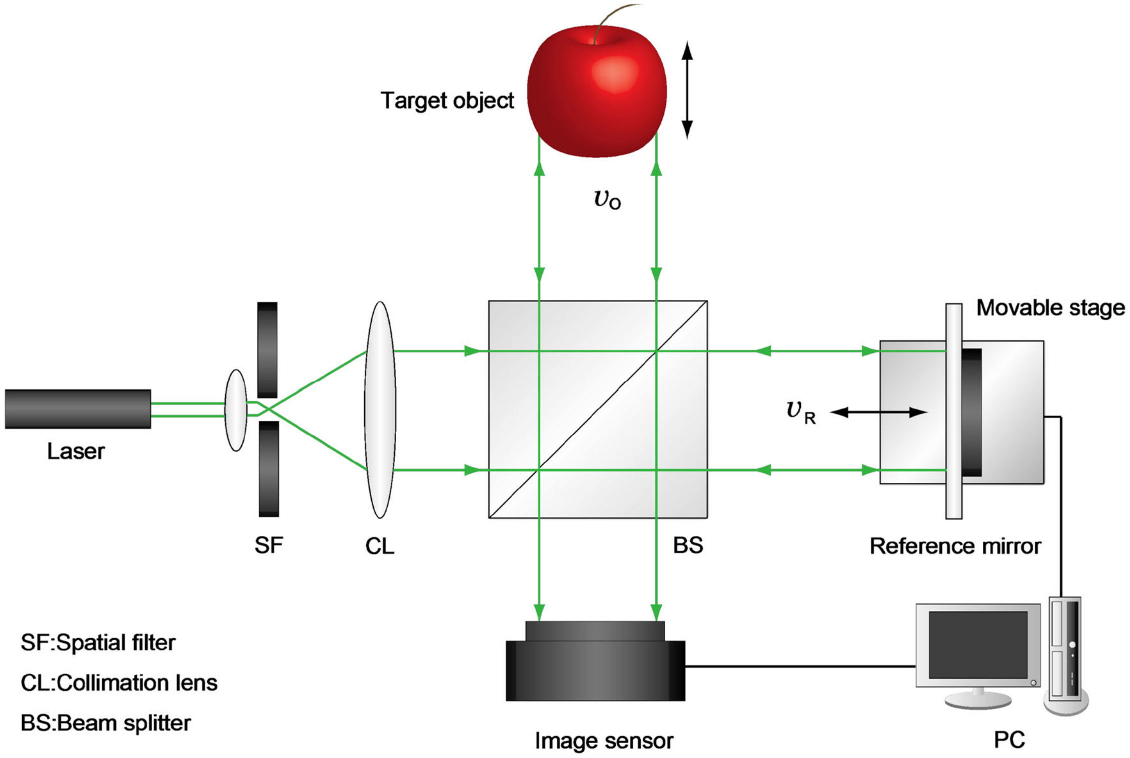

Fig. 1 Experimental setup for Doppler phase-shifting digital holography

here,

$$\begin{aligned}\Delta\phi(x,y) &= \phi_{\mathrm{O}}(x,y,t) - \phi_{\mathrm{R}}(x,y,t)\\ \Delta\omega(t) &= |\omega_{\mathrm{O}}(t) - \omega_{\mathrm{R}}(t)| \cong 2\omega_0|\upsilon_{\mathrm{O}}(t) - \upsilon_{\mathrm{R}}(t)|/c\end{aligned} \tag{6}$$

The Fourier transformation of Eq. (5) in the time domain is

$$\begin{aligned}\mathcal{F}_t(I)(x,y,\omega_{\mathrm{b}}) = {} & \left[a_{\mathrm{O}}^2(x,y) + a_{\mathrm{R}}^2\right]\delta(\omega_{\mathrm{b}})\\ & + a_{\mathrm{O}}(x,y)a_{\mathrm{R}}A_1(\omega_{\mathrm{b}})\exp\{\mathrm{i}[\Delta\phi(x,y) + B_1(\omega_{\mathrm{b}})]\}\\ & + a_{\mathrm{O}}(x,y)a_{\mathrm{R}}A_2(\omega_{\mathrm{b}})\exp\{-\mathrm{i}[\Delta\phi(x,y) + B_2(\omega_{\mathrm{b}})]\}\end{aligned} \tag{7}$$

where A_1, A_2, B_1 and B_2 are the arbitrary functions of the beat frequency ω_{b}. If the angular frequency of the object light is higher than that of the reference light, the +1st order diffraction is obtained in the second term. If the spectrum of each term can be split, the phase difference $\Delta\phi(x, y)$ between the object light and the reference light is easily obtained.

The discrete Fourier transformation of the image recorded by the image sensor of the finite size is obtained as:

$$\begin{aligned}\mathrm{DFT}[I]\left(x,y,\nu_{\mathrm{b},m}\right) &= \sum_{n=0}^{N-1} I(x,y,t_n)\exp\left(-\mathrm{i}2\pi t_n \nu_{\mathrm{b},m}\right)\\ t_n = n\Delta t,\ \nu_{\mathrm{b},m} &= \frac{m}{N\Delta t}(m = 0,1,2,\cdots,N-1)\end{aligned} \tag{8}$$

If the +1st order diffraction spectrum of the beat frequency ν_{b} only is extracted from Eq. (8), the complex amplitude of an object can be calculated very accurately from the second term in Eq. (7).

3 Measurement of shape and velocity

Basically the Doppler phase-shifting is invented to measure the object profile accurately. But, as the frequency of the Doppler beat is proportional to the velocity difference between the object light and the reference light, the longitudinal velocity of an object can be measured by the beat frequency. Thus, the longitudinal velocity of a concave mirror travelling at a constant speed is directly measured by a Doppler phase-shifting holography in this study.

Ar-ion laser of 514 nm is used as the light source, which is introduced to an object lens and then shaped by a spatial filter and finally made parallel by a collimator lens of $f = 100$ mm. A concave mirror of the radius of curvature is $R = 30$ m is traversed at $\upsilon_{\mathrm{O}} = 100$ µm/s by the voice coil motor and the reference mirror is at a rest, $\upsilon_{\mathrm{R}} = 0$ µm/s. The interference images are recorded by a high-speed camera whose resolution is 1024 × 1024 and its frame rate is 2000 fps with a shutter speed of 1/70,000 s. Figure 2a shows the time series intensity fluctuation of the central pixel of the 512 images recorded by a high-speed camera and the Fig. 2b is its spectrum. Doppler beat peaks can clearly be seen at a frequency of $\upsilon_{\mathrm{b}} = \omega_{\mathrm{b}}/2\pi = 386.7$ Hz, which corresponds to $\upsilon_{\mathrm{O}} = 99.4$ µm/s by Eq. (6) with an error of 0.6 % of the given speed.

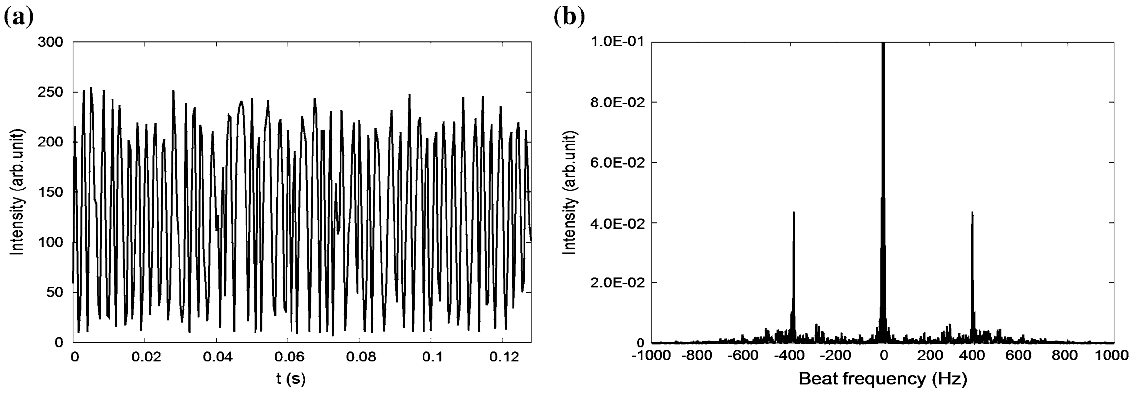

Fig. 2 **a** Intensity of a pixel at the image center and **b** Spectrum obtained from 512 images

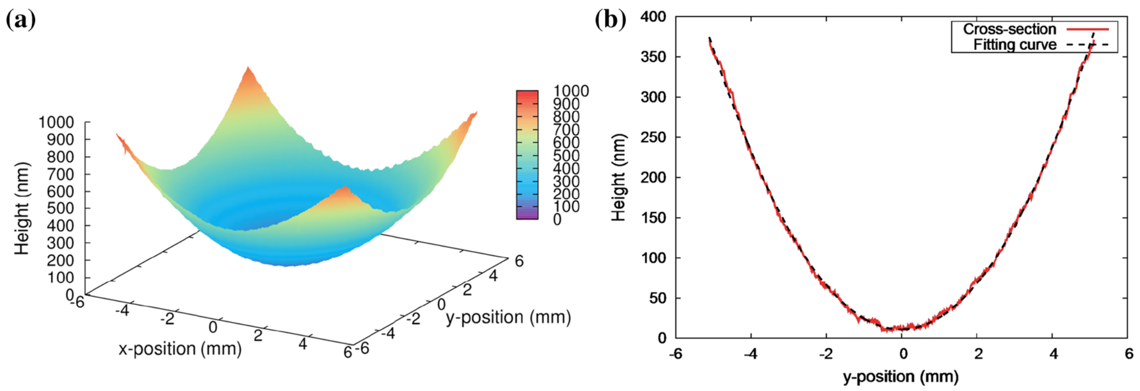

Fig. 3 **a** Measured shape of concave mirror and **b** Central cut by the *y*-axis and a curve fit

Now it has been proved that a Doppler phase-shifting holography can be used for the measurement of the longitudinal velocity. Moreover, the Fig. 3 shows the 3D and 2D plots of the surface profile of the concave mirror reproduced by digital holography and the rms error to the $R = 30$ m profile is only 3.27 nm. This is the evidence that the present technique is capable of measuring the three-dimensional position and the longitudinal velocity simultaneously.

4 Conclusion

Presently, a new three-dimensional velocity measuring technique based on a Doppler phase-shifting holography has been developed. It can measure the longitudinal velocity of an object very accurately by the single camera observation. This method can be adapted to the particle flows and then the three-dimensional particle velocities can be measured simultaneously. It will sure to contribute to the development of micro- and nano-fluidic devices.

Acknowledgments A part of this work has been supported by the Center for Optical Research and Education (CORE) at Utsunomiya University, the Project for Bio-imaging and Sensing at Utsunomiya University and the JSPS KAKENHI Grant Number 25420147. All supports are greatly acknowledged.

References

Kikuchi Y,Barada D,Kiire T,Yatagai T (2010) Doppler phase-shifting digital holography and its application to surface shape measurement. Opt Lett, 35(10)
Schnars U, Jueptner W (2005) Digital holography digital hologram recording, numerical reconstruction, and related techniques. Springer